Space–time discretization: the way to the fundamental element without a determined form.

S.B. Afanas'ev
St.–Petersburg, Russia

Abstract

The concept of the random discretization of the space–time is suggested. It is the way to consistent compatible synthesis of quantum and relativistic principles and principle of geometrization. The basic idea of this concept is physical reality of the finite sizes fundamental element of the quantized space–time. The flat space–time with random discretization is described as the probability measure space with the set of all possible discretizations of the flat continual space–time as the set of points. The probability measure can depend on the geometric parameters of discretizations (a number of regions of a discretization, their volumes, areas etc.). In this concept the fundamental length can be defined as the average value of the linear size of a fundamental element. In this concept the "particle" quantum and the space-time quantum are identical.

The synthesis of relativistic, quantum and geometrization principles is one of the central problem of modern physics. Below this set of principles (quantum, relativistic and principle of geometrization) is conventionally named QRG principles. The approaches to their consistent compatible realizing are found both on the way of the solution of the quantum gravity problem and quantization of the space–time [1] and in the creation of the united theory of fundamental interactions and the geometrized particle theory. In the last theories (in particular, in the superstring theory [2]) the particles are represented as the excited states of some extended objects. In this work an approach, in which the particles can be described as the excited states of n–dimensional fundamental elements of the space–time, is suggested. This approach is based on the representation of discrete structure of the quantized space–time (QST) as the set of finite size fundamental elements. We suppose that these fundamental elements have the property of physical reality. In other words, the physical space–time is the one with the discrete structure, but not continual space–time that consist of points, i.e. fundamental elements without sizes and structure. The continual space–time is the limiting case of the discrete space–time with the zeroth value of the fundamental length and has the sense of mathematical abstract.

The consideration of the space–time fundamental element as the physical reality element and the principles of quantum description of microobjects necessarily lead to abandonment of determined form and sizes of a fundamental element (since they are variables describing quantum object and cannot have exactly determined values) and representation of QST as the probability measure space with the set of all possible discretizations of the continual space–time as the set of points. Therefore this concept is properly named the concept of the random discretization of the space–time. At first glance this concept is more compatible consistent realizing of QRG principles than both the known approaches to the space–time discrete structure description and the existed geometrized theories of particles. Thus in the superstring theory the "particle" quantum (i.e. all particles are excited states of this quantum) is not identical to the quantum of the space–time, and the string as quantum geometrical object can be defined as the space quantum only. In the suggested concept the "particle" quantum is the fundamental element of the quantized space–time, or the quantum of the space–time. On the other hand, this concept contain many other discrete space–time models [3] (the lattice space–time, the Regge simplicial space–time etc.) as special cases of investigated object with the special choices of the probability measure.

In this work the basic ideas and representations of the concept of the random discretization are discussed. They are concerned with the semiclassical method of description of discrete structure of the space–time. The questions about nature of particles as the excitations of QST is briefly discussed in the conclusion. In this work the case of the space–time with the average values equal over all discretized manifold is analyzed.

Consider the semiclassical description of the discrete structure of QST in the framework of the concept of the random discretization. Below the geometric base of semiclassical description of the flat space–time (or the space) is discussed. Obviously, difference between the space–time and the space is not important as long as the metric relations are not considered.

The manifold of the flat space with random discretization (briefly - random discretized flat manifold) is the probability measure space with the set of all possible discretizations of $M \subset \mathcal{R}^n$ as the set of points. The probability measure defined on this set can depend on the scalars of discretizations, i.e. a number of regions of a discretization, their volumes, areas of surface etc. Below we suppose that the measure on the set of discretizations can be defined. Correct definition of the probability measure on the set of all $\mathcal{R}^n$ – discretizations has some difficulties, and the flat space with random discretization can be defined as the set of all random discretized flat manifolds (or representative set of manifolds, for example, all spaces with the set of discretizations of open sphere). Denote the set of discretizations $D(M)$ and the discrete space with random discretization $RanD(M, d\mu)$, where M is the discretized manifold of $\mathcal{R}^n$, $d\mu$ is the probability measure. The probability measure $d\mu$ is represented in the form

$$d\mu = \frac{1}{I} \cdot \mu_p d\sigma, \tag{1}$$

where $d\sigma$ is the measure on the set of discretizations, μ_p — the factor characterized non-equality of the probabilities of discretizations, $I = \int\limits_{D(M)} \mu_p \, d\sigma$ The sense of notation μ_p is following: different values of probability density of discretizations are caused by the physical properties of excitations of the space–time, and it can be says that μ_p is the physical factor of the probability measure.

Below only the measures dependent on the number of regions N, their volumes $\{V_i\}_{i=1}^N$ and their surface areas $\{S_i\}_{i=1}^N$ are considered. In the general case μ_p is represented as the sum of series

$$\mu_p = \sum A_{\alpha\{\beta\}\{\gamma\}} N_\alpha V_1^{\beta_1} ... V_N^{\beta_N} S_1^{\gamma_1} ... S_N^{\gamma_N} \tag{2}$$

The average values of N, V and S are defined in the following manner:

$$< N >= \int\limits_{D(M)} N d\mu \tag{3}$$

$$< V >= \int\limits_{D(M)} \bar{V}^{\{l\}} d\mu^{\{l\}} \tag{4}$$

where $\bar{V}^{\{l\}} = \frac{1}{N} \sum_{i=1}^N V_i^{\{l\}}$, $\{l\}$ is identified the discretizations

$$< S >= \int\limits_{D(M)} \bar{S}^{\{l\}} d\mu^{\{l\}} \tag{5}$$

where $\bar{S}^{\{l\}} = \frac{1}{N} \sum_{i=1}^N S_i^{\{l\}}$ Thus the average values of V and S are calculated by double–averaging: over the regions of a discretization and over discretizations.

Consider also the subset of the probability measures from the set (2) that are represented in the form:

$$\mu_p = f(\sum_{i=1}^N A_{\alpha\beta\gamma} N^\alpha V_i^\beta S_i^\gamma) \tag{6}$$

Obviously, basic interest is caused by the measures for which the average values $< N >$, $< N >$ and $< S >$ are finite.

It can be supposed that these measures must satisfy following conditions

$$\lim_{N \to 1} \mu_p = \mu_1 \tag{7}$$

$$\lim_{N \to \infty} \mu_p = 0 \tag{8}$$

These conditions are necessary because both the finiteness of values of μ_p for discretizations with the small number of regions N and the zeroth limit of μ_p with $N \to \infty$ are required for the finiteness of integrals by type

$$< N >= \int N d\mu \tag{9}$$

The problem, are the conditions (7, 8) sufficient for the finiteness of values $< N >$, $< V >$ and $< S >$, is open. It is not excluding that the finiteness of values $< S >$ requires the satisfaction of special conditions for values of the probability measure for discretizations that contain the regions with the large values S. It can be suggested several dependencies of $\mu_p(V, S, N)$ that are satisfied the conditions (7) and (8)

$$\mu_p = C\, exp(-\, A \sum_i S_i) \tag{10}$$

$$\mu_p = C\, exp(-\, A \sum_i \frac{S_i}{V_i}) \tag{11}$$

$$\mu_p = C\, exp(-\, A \sum_i \frac{S_i}{V_i} - B \sum_i \frac{V_i}{S_i}) \tag{12}$$

Besides the problem of dependence of average values $< V >$, $< S >$ and $< N >$ on the size of a discretized manifold has the deep sense. Obviously, this dependence must satisfy some requirements for the measures interesting for the description of the physical space–time. It seems likely that this problem is connected with the conformal invariance in the investigated discrete space. At first glance the condition of independence (or weak dependence) of average values of V and S on the continual manifold size is most reasonable condition.

Thus the considered probability measure space is realizing of discrete space (space–time) structure with the finite value of the fundamental length (with the supposition of existence of the set of probability measures that give the finite values of N, V and S). It is noted that earlier investigated concepts of space–time discretization are the special cases of considered object. Thus the lattice space–time is the special case of $RanD(M, d\mu)$ with the following choice of the probability measure: $\mu = \mu_0$ for discretizations with the lattice as the set of boundaries of regions, and $\mu = 0$ for other discretizations. The Regge simplicial space–time is the special case of considered space with the probability measure different from zero for discretizations by simplexies.

In this concept the fundamental length has the meaning some average linear size of regions of discretizations. It can be suggested three geometric parameters by this type:

$$l_I =< 2n \frac{V}{S} > \tag{13}$$

$$l_{II} =< V^{1/n} > \tag{14}$$

$$l_{III} =< V >^{1/n} \tag{15}$$

where n is the dimensionality of the space–time, and the average values are defined by the method of double–averaging (see (4), (5)).

Average values of the geometrical variables can be calculated with using of the construction that is analogous to the continual (functional) integral. This construction can be considered as the generalization of the Feynman integral over trajectories and the string world surfaces integral.

In conclusion, it is some words about development of considered concept. Considered method of discretization of the flat space–time is the first step to realizing of synthesis of relativistic, quantum and geometrization principles. This method of discretization of the space–time gives the way to the introducing of mathematical operations on the set of regions of discretizations (different from the continual space–time operations). This way can be more consistent realizing of three basic principles than the string and brane theories and other concepts of the space–time discretizations by the following causes:

1) This space–time is compatible quantized (in this context, disretized) automatically (connection with the algebraic problem of space-time quantization see below), and the continual space–time don't play the role of method of space–time description as it is in the superstring theory;

2) This space–time description method allow to introduce the operations on the set of fundamental elements (regions of discretizations), and principally property of relativistic invariance can be formulated in terms of quantized (discrete) space–time;

3) Particles can be considered as excited states of fundamental elements of the discrete space–time, and different particles are described by the different dependence of the probability measure on the parameters of discretizations. Last conclusion is in the agreement with the form of general construction of the functional integral, in which μ_p is coincided with factor $exp(-S)$, where S is an action;

4) In this concept the "particle" quantum is identical to the space-time quantum. Thus this concept meet the principle of minimal number of basic object on the most fundamental level.

It is noted the following problems for the development of this concept:

1) Formulation of $RanD(M, d\mu)$ – coordinate– dependent probability space (space–time) in the framework of semiclassical description considered in this article;

2) Introduction of the mathematical operations on $RanD(M, d\mu)$. This approach allow to describe space–time in internal terms of invariance and transformations;

3) Research of the random discretization of the curved space–time. In this concept all tensors are the random ones, and equations and equalities of curved discrete space–time have the probability sense;

4) Connection between the discrete structure of the space–time and the algebraic description of the quantized space–time (commutation relations, introducing of operators of creation and destruction etc.). The solution of this problem is required introduction of the fundamental elements of quantized space–time as the elementary units of not $\mathcal{R}^n$, but the all investigated probability measure space, i.e. introduction of elementary units of the set of all discretizations. In this approach operators of QST are described the fundamental elements of the one;

5) Relation between calculations over the set of discretizations and the ones over the topology (set of all possible subsets of $\mathcal{R}^n$). This problem is connected with the problem of correspondence of the probability measures defined on the set of discretizations and on the topology [4];

6) Formulation of the compatible quantum description of the discrete structure of the space-time, i.e. definition of the wave function on the set of discretizations and finding of basic equations and fundamental properties of this wave function.

References

[1] Treder H.-J. Fortsh. Phys. **11** 81 (1963);
Hawking S. W. In: General relativity. Cambridge University Press, Cambridge, 1979;
Hawking S. W. Nucl. Phys. **144**, 349 (1978);
Wheeler J. A. Geometrodynamics. Academic Press, New York, 1962;
Wheeler J. A. In: Relativity Groups and Topology, eds. B.S. and C.M. De Witt. Gordon and Breach, New York, 1963;
Snyder H. S. Phys. Rev. **71**, 38 (1947);
Rovelli C. gr-qc/9903045.

[2] Green M. B., Schwarz J. H., Witten E. **Superstring theory.** Cambridge Univ. Press, Cambrigde, 1987.

[3] Darling B., Zilsel P. Phys. Rev. **91**, 1252 (1953);
Shiff L. Phys.Rev. **92**, 766 (1953);
Darling B. Phys. Rev. **80**, 460 (1950);
Ambarzumian V., Iwanenko D. Z. Phys. **64**, 563 (1930);
Miller D., Milton K.A., Siegemund–Broka S. Phys. Rev. **D46**, 806 (1992);
Miller D. Nucl. Phys. Proc. Suppl. **B34**, 774 (1994);
Bender C. M. et al Phys. Rev. **D31**, 383 (1985);
Chiu T. W. Nucl. Phys. Proc. Suppl. **B34**, 599 (1994);
Regge T. Nuov. Cim. **19**, 558 (1965);
Williams R. M., Tuckey P. A. Class. Quant. Grav. **9**, 1409 (1992);

[4] Balachandran A. P., Bimonte G., Ercolessi E. et al Nucl. Phys. Proc. Suppl. **37C**, 20 (1995);
hep-th/9403067.